

# Mathematics

The Department of Mathematics offers one master's degree in mathematics and one doctoral degree in mathematics. The areas of study for mathematics include algebra, algebraic geometry, real and complex analysis, differential geometry, and topology. Because it is difficult to make up coherent programs for students entering in the middle of the year, students are ordinarily admitted only in the fall.

When they first arrive, graduate students have the opportunity to share common concerns and to become acquainted. One of the most attractive features of our program is the friendly and supportive atmosphere that develops among our graduate students. Advanced courses in the Washington University Mathematics department can build on the common background shared by all students. As a result, these courses are richer and nearer to the level of PhD work than typical advanced courses.

Students typically complete the PhD program in five years, and those students may expect up to five years of support. Continuation of support each year is dependent upon normal progress toward the degree and the satisfactory performance of duties. A student who comes to Washington University with advanced preparation may finish in less time. On the other hand, some students find that it is advisable for them to take preparatory math courses before attempting the qualifying courses. In special cases, the time schedule may be lengthened accordingly. Each student should plan to develop a close relationship with their thesis advisor so that the advisor may have a realistic idea of the student's progress.

Graduate study in mathematics is not for everyone. Entering students usually find that the time and effort required to succeed goes well beyond anything they encountered as undergraduates. Success requires both ample mathematical ability and the determination to grapple with a subject for many days or weeks until the light of understanding shines through, and the experience can be daunting. Those who continue in their studies are largely those for whom the pleasure of attaining that understanding more than compensates for the required effort. For such persons, the life of a mathematician can be richly rewarding.

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## Faculty

### Chair

#### John Shareshian

Professor

PhD, Rutgers University

Algebraic and topological combinatorics

### Director of Graduate Studies

#### Gregory Knese

Professor

PhD, Washington University

Complex function theory; operators; harmonic analysis

### Director of Undergraduate Studies

#### Ari Stern

Professor

PhD, California Institute of Technology

Geometric numerical analysis; computational mathematics

### Associate Director of Undergraduate Studies

#### Blake Thornton

Teaching Professor

PhD, University of Utah

Geometric topology

### Department Faculty

#### Roya Beheshti Zavareh

Professor

PhD, Massachusetts Institute of Technology

Algebraic geometry

#### Alan Chang

Assistant Professor

PhD, University of Chicago

Geometric measure theory; harmonic analysis

#### Quo-Shin Chi

Professor

PhD, Stanford University

Differential geometry

#### Lawrence Conlon

Emeriti Professor

PhD, Harvard University

Differential topology

#### Aliakbar Daemi

Assistant Professor

PhD, Harvard University

Gauge theory; low-dimensional topology; symplectic geometry

#### Laura Escobar Vega

Associate Professor

PhD, Cornell University

Combinatorics; algebraic geometry

#### Renato Feres

Professor

PhD, California Institute of Technology

Differential geometry; dynamical systems

**Steven Frankel**

Associate Professor  
PhD, University of Cambridge  
Geometric topology; dynamics

**Ron Freiwald**

Emeriti Professor  
PhD, University of Rochester  
General topology

**Andrew Walton Green**

William Chauvenet Postdoctoral Lecturer  
PhD, Clemson University  
Harmonic analysis; partial differential equations

**Gary R. Jensen**

Emeriti Professor  
PhD, University of California, Berkeley  
Differential geometry

**Silas Johnson**

Senior Lecturer  
PhD, University of Wisconsin–Madison  
Algebraic number theory; arithmetic statistics

**Matt Kerr**

Professor  
PhD, Princeton University  
Algebraic geometry; Hodge theory

**Steven G. Krantz**

Professor  
PhD, Princeton University  
Several complex variables; geometric analysis

**N. Mohan Kumar**

Emeriti Professor  
PhD, Bombay University  
Algebraic geometry; commutative algebra

**Wanlin Li**

Assistant Professor  
PhD, University of Wisconsin–Madison  
Number theory; arithmetic geometry

**Henri Martikainen**

Associate Professor  
PhD, University of Helsinki, Finland  
Harmonic analysis; geometric measure theory

**John E. McCarthy**

Spencer T. Olin Professor of Mathematics  
PhD, University of California, Berkeley  
Analysis; operator theory; one and several complex variables

**Minh Nguyen**

Postdoctoral Lecturer  
PhD, University of Arkansas  
Gauge theory; low dimensional topology

**Charles Ouyang**

Assistant Professor  
PhD, Rice University  
(Higher) Teichmüller theory; Riemann surfaces; harmonic maps and minimal surfaces

**Martha Precup**

Associate Professor  
PhD, University of Notre Dame  
Applications of Lie theory to algebraic geometry and the related combinatorics

**Donsub Rim**

Assistant Professor  
PhD, University of Washington  
Applied mathematics

**Rachel Roberts**

Elinor Anheuser Professor of Mathematics  
PhD, Cornell University  
Low-dimensional topology

**Richard Rochberg**

Emeriti Professor  
PhD, Harvard University  
Complex analysis; interpolation theory

**Angel Roman**

Postdoctoral Lecturer  
PhD, Pennsylvania State University  
Representation theory; operator algebras

**Jesus Sanchez**

Postdoctoral Lecturer  
PhD, Pennsylvania State University  
Noncommutative index theory; cyclic cohomology; spin Riemannian geometry; high-dimensional gauge theory

**Karl Schaefer**

Lecturer  
PhD, University of Chicago  
Algebraic number theory

**Jack Shapiro**

Emeriti Professor  
PhD, City University of New York  
Algebraic K-theory

**Edward Spitznagel**

Emeriti Professor  
PhD, University of Chicago  
Statistics; statistical computation; application of statistics to medicine

**Yanli Song**

Associate Professor  
PhD, Pennsylvania State University  
Noncommutative geometry; symplectic geometry; representation theory

### **Xiang Tang**

Professor  
PhD, University of California, Berkeley  
Symplectic geometry; noncommutative geometry; mathematical physics

### **Joel Villatoro**

Postdoctoral Lecturer  
PhD, University of Illinois at Urbana-Champaign  
Differential geometry; Poisson geometry; singular spaces

### **Brett Wick**

Professor  
PhD, Brown University  
Complex analysis; harmonic analysis; operator theory; several complex variables

### **Mladen Victor Wickerhauser**

Professor  
PhD, Yale University  
Harmonic analysis; wavelets; numerical algorithms for data compression

### **Edward N. Wilson**

Emeriti Professor  
PhD, Washington University  
Harmonic analysis; differential geometry

### **David Wright**

Emeriti Professor  
PhD, Columbia University  
Affine algebraic geometry; polynomial automorphisms

### **Jay Yang**

Postdoctoral Lecturer  
PhD, University of Wisconsin–Madison  
Commutative algebra; algebraic geometry

## **Degree Requirements**

- Mathematics, AM
- Mathematics, PhD

## **Courses**

Visit online course listings to view semester offerings for L24 Math.

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### **L24 Math 501C Theoretical Physics**

The first part of a two-semester course reviewing the mathematical methods essential for the study of physics. Theory of functions of a complex variable, residue theory; review of ordinary differential equations; introduction to partial differential equations; integral transforms. Prerequisite: undergraduate differential equations (Math 217), or permission of instructor.  
Same as L31 Physics 501  
Credit 3 units.

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### **L24 Math 5021 Complex Analysis I**

An intensive course in complex analysis at the introductory graduate level. Math 5021 and Math 5022 form the basis for the Ph.D. qualifying exam in complex analysis. Prerequisite: Math 4111, 4171 and 4181, or permission of the instructor.  
Credit 3 units.

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### **L24 Math 5022 Complex Analysis II**

Continuation of Math 5021. Prerequisite, Math 5021 or permission of instructor.  
Credit 3 units.

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### **L24 Math 502C Methods of Theoretical Physics II**

Continuation of Phys 501. Introduction to function spaces; self-adjoint and unitary operators; eigenvalue problems, partial differential equations, special functions; integral equations; introduction to group theory. Prerequisite: Phys 501, or permission of instructor.  
Same as L31 Physics 502  
Credit 3 units.

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### **L24 Math 5031 Algebra I**

An introductory graduate level course on the basic structures and methods of algebra. Detailed survey of group theory including the Sylow theorems and the structure of finitely generated Abelian groups, followed by a study of basic ring theory and the Galois theory of fields. Math 5031 and Math 5032 form the basis for the Ph.D. qualifying exam in algebra. Prerequisite: Math 430 or the equivalent, or permission of the instructor.  
Credit 3 units.

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### **L24 Math 5032 Algebra II**

Continuation of Math 5031. Prerequisite: Math 5031 or permission of instructor.  
Credit 3 units.

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### **L24 Math 5041 Geometry I**

Introductory graduate level course including differential calculus in  $n$ -space; differentiable manifolds; vector fields and flows; differential forms and calculus on manifolds; elements of Lie groups and Lie algebras; Frobenius theorem; elements of Riemannian geometry. Math 5041 and Math 5042 (or 5043) form the basis for the Ph.D. qualifying exam in geometry / topology. Prerequisites: Math 4121, 429, and 4181, or permission of the instructor.  
Credit 3 units.

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### **L24 Math 5042 Geometry II**

Continuation of Math 5041. Math 5042 and Math 5043 are offered in alternate spring semesters as a sequel to Math 5041. Prerequisite: Math 5041 or permission of instructor.  
Credit 3 units.

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### **L24 Math 5045 Geometry/Topology I: Algebraic Topology**

An introductory graduate-level course in algebraic topology, including fundamental groups, covering spaces, homology, and cohomology. Prerequisites: undergraduate courses in abstract algebra and point-set topology or permission from the instructor. Replaces 5043.  
Credit 3 units.

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### **L24 Math 5046 Geometry/Topology II: Differential Topology**

An introductory graduate-level course in the topology of smooth manifolds and vector bundles. Prerequisites: Math 5045 (GT I: Algebraic Topology) or permission from the instructor. Replaces 5041.

Credit 3 units.

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**L24 Math 5047 Geometry/Topology III: Differential Geometry**

An introductory graduate-level course in the geometry of smooth manifolds and vector bundles. Prerequisites: Math 5046 (Geometry/Topology II: Differential Topology) or permission from the instructor. Replaces 5042.  
Credit 3 units.

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**L24 Math 5051 Measure Theory and Functional Analysis I**

Introductory graduate level course including the theory of integration in Euclidean and abstract spaces, and an introduction to the basic ideas of functional analysis. Math 5051 and Math 5052 form the basis for the Ph.D. qualifying exam in real analysis. Prerequisites: Math 4111, 4171, and 4181, or permission of the instructor.  
Credit 3 units.

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**L24 Math 5052 Measure Theory and Functional Analysis II**

Continuation of Math 5051. Prerequisite: Math 5051 or permission of instructor.  
Credit 3 units.

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**L24 Math 510 Introduction to Fourier Series and Integrals**

The basic theory of Fourier series and Fourier integrals including different types of convergence. Applications to certain differential equations. Prerequisites: Math 4111 or permission of instructor. Same as L24 Math 410  
Credit 3 units. A&S IQ: NSM

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**L24 Math 5101 Introduction to Analysis**

The real number system and the least upper bound property; metric spaces (completeness, compactness, and connectedness); continuous functions (in  $\mathbb{R}^n$ ; on compact spaces; on connected spaces);  $C(X)$  (pointwise and uniform convergence; Weierstrass approximation theorem); differentiation (mean value theorem; Taylor's theorem); the contraction mapping theorem; the inverse and implicit function theorems. Prerequisite: Math 310 or permission of instructor. Same as L24 Math 4111  
Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 5102 Introduction to Lebesgue Integration**

Riemann integration; measurable functions; measures; Lebesgue measure; the Lebesgue integral; integrable functions;  $L^p$  spaces; modes of convergence; decomposition of measures; product measures. Prerequisite: Math 4111 or permission of the instructor. Same as L24 Math 4121  
Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 515 Theory of Partial Differential Equations I**

A rigorous mathematical study of topics in partial differential equations. Prerequisites: Math 5051 and Math 5052 or equivalent. Some knowledge of complex analysis will also be useful. No prior knowledge of partial differential equations is required.  
Credit 3 units.

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**L24 Math 516 Theory of Partial Differential Equations II**

Credit 3 units.

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**L24 Math 5160 Complex Variables**

Analytic functions, elementary functions and their properties, line integrals, the Cauchy integral formula, power series, residues, poles, conformal mapping and applications. Prereq: Math 310 and (Math 318 or Math 4111), or permission of instructor. Same as L24 Math 416  
Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 517 Hilbert Spaces I**

Credit 3 units.

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**L24 Math 518 Hilbert Spaces II**

Credit 3 units.

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**L24 Math 519 Harmonic Analysis I**

Credit 3 units.

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**L24 Math 520 Harmonic Analysis II**

Credit 3 units.

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**L24 Math 5201 Topology I**

An introduction to the most important ideas of topology. Course includes necessary ideas from set theory, topological spaces, subspaces, products and quotients, compactness and connectedness. Some time is also devoted to the particular case of metric spaces (including topics such as separability, completeness, completions, the Baire Category Theorem, and equivalents of compactness in metric spaces). Prerequisite: Math 4111 or permission of instructor. Same as L24 Math 4171  
Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 5202 Topology II**

A continuation of Math 4171 featuring more advanced topics in topology. The content may vary with each offering. Prerequisite: Math 4171, or permission of instructor. Same as L24 Math 4181  
Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 521 Topics in Complex Variables I**

Credit 3 units.

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**L24 Math 522 Topics in Complex Variables II**

Credit 3 units.

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**L24 Math 523 Topics in Analysis**

Analytic Combinatorics  
Credit 3 units.

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**L24 Math 523C Information Theory**

Discrete source and channel model, definition of information rate and channel capacity, coding theorems for sources and channels, encoding and decoding of data for transmission over noisy channels. Corequisite: ESE 520. Same as E35 ESE 523  
Credit 3 units. EN: BME T, TU

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**L24 Math 527 Functional Analysis I**

Credit 3 units.

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**L24 Math 528 Topics in Functional Analysis II**

Credit 3 units.

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**L24 Math 5301 Linear Algebra**

This course is an introduction to the linear algebra of finite-dimensional vector spaces. It includes systems of equations, matrices, determinants, inner product spaces, and spectral theory. Prerequisite: Math 310 or permission of instructor. Math 309 is not an explicit prerequisite, but students should already be familiar with such basic topics from matrix theory as matrix operations, linear systems, row reduction, and Gaussian elimination. (Material on these topics in early chapters of the text will be covered very quickly.)

Same as L24 Math 429

Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 5302 Modern Algebra**

Introduction to groups, rings, and fields. Includes permutation groups, group and ring homomorphisms, field extensions, connections with linear algebra. Prerequisite: Math 310, Math 429 or permission of the instructor.

Same as L24 Math 430

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 535 Topics in Combinatorics**

Credit 3 units.

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**L24 Math 537 Topics in Algebra I**

Credit 3 units.

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**L24 Math 538 Topics in Algebra II**

Credit 3 units.

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**L24 Math 539 Topics in Algebraic Geometry**

Selected topics in algebraic geometry.

Credit 3 units.

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**L24 Math 5392 Topics in Algebraic Geometry II**

Credit 3 units.

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**L24 Math 541 Topics in Applied Mathematics**

Topic and prerequisites vary with each offering of the course.

Credit 3 units.

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**L24 Math 543 Geometry and Manifold Theory I**

Credit 3 units.

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**L24 Math 544 Topics in Geometry and Manifold Theory II**

Credit 3 units.

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**L24 Math 545 Topics in Riemannian Geometry I**

Credit 3 units.

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**L24 Math 546 Topics in Riemannian Geometry II**

Credit 3 units.

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**L24 Math 547 Topics in Geometry**

An introduction to Geometric Group Theory, concentrating on the theory of hyperbolic groups and group boundaries.

Credit 3 units.

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**L24 Math 550 Topics in Number Theory: Analytic Number Theory**

The course will develop analytic methods for problems which occur in algebraic number theory and algebraic geometry. We will consider Riemann zeta function, Dirichlet L-functions, multiple zeta functions, multiple Dirichlet L-functions (according to Manin), polylogarithms, reciprocity laws on curves and surfaces and multiple Dedekind zeta functions. Prerequisite: Permission of Instructor

Credit 3 units.

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**L24 Math 5501 Numerical Applied Mathematics**

Computer arithmetic, error propagation, condition number and stability; mathematical modeling, approximation and convergence; roots of functions; calculus of finite differences; implicit and explicit methods for initial value and boundary value problems; numerical integration; numerical solution of linear systems, matrix equations, and eigensystems; Fourier transforms; optimization. Various software packages may be introduced and used. Prerequisites: Math 217 or 312, Math 309, Math 310 and CSE 131 (or other computer background with permission of the instructor).

Same as L24 Math 449

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 5502 Topics in Applied Mathematics**

Topic may vary with each offering of the course. Prerequisite: CSE 131 and, Math 449, or permission of the instructor.

Same as L24 Math 450

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 5560 Topics in Financial Mathematics**

An introduction to the principles and methods of financial mathematics, with a focus on discrete-time stochastic models. Topics include no-arbitrage pricing of financial derivatives, risk-neutral probability measures, the Cox-Ross-Rubenstein and Black-Scholes-Merton options pricing models, and implied volatility. Prerequisites: Math 233, Math 3200, Math 310 or permission of instructor.

Same as L24 Math 456

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 560 Topics in Topology**

Credit 3 units.

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**L24 Math 563 Topics in Lie Groups and Algebras I**

Credit 3 units.

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**L24 Math 564 Topics in Lie Groups and Algebras II**

Credit 3 units.

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**L24 Math 565 Algebraic Topology I**

Credit 3 units.

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**L24 Math 566 Algebraic Topology II**

Credit 3 units.

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**L24 Math 567 Several Complex Variables I**

Credit 3 units.

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**L24 Math 568 Several Complex Variables II**

Credit 3 units.

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**L24 Math 569 Topics in Homological Algebra**

Credit 3 units.

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**L24 Math 571 Topics in Combinatorics: Spectral Graph Theory**

Credit 3 units.

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**L24 Math 572 Topics in Set Theory and Logic**

Credit 3 units.

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**L24 Math 581 Introduction to Computational Analysis**

Credit 3 units.

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**L24 Math 583 Higher Methods of Computational Analysis**

Credit 3 units.

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**L24 Math 590 Research**

Credit variable, maximum 3 units.

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**L24 Math 595 Seminar**

Credit variable, maximum 3 units.

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**L24 Math 596 Seminar**

Credit variable, maximum 3 units.

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**L24 Math 597 Teaching Seminar**

Principles and practice in the teaching of mathematics at the college and university level. Prerequisite: graduate standing, or permission of instructor.

Credit 1 unit.

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**L24 Math 598 Mathematical Professional Development**

This course includes topics on professional development and responsible conduct of research. Prerequisites: none.

Credit 1 unit.

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**L24 Math 883 Master's Continuing Student Status**

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