

# Mathematics

Mathematics has always held a central position in the liberal arts, and over time, it has also come to play an important role in more and more aspects of our lives. Mathematical analysis and modeling are involved in many areas far beyond the traditional association of mathematics with the physical sciences and engineering. This fact is reflected in the diversity of the students who study at least some mathematics during their time at Washington University — students who recognize the importance of quantitative skills in a world that is becoming more and more technological.

Students major in mathematics for many reasons. Some are planning academic careers in mathematics that involve teaching or research. Others plan to work as actuaries or at other jobs in industry or government. Some plan careers in secondary education. Many majors do not intend to become "mathematicians" at all but simply realize that quantitative training is a valuable asset in many kinds of careers; often, work in mathematics is meant to complement their study in other areas. Other majors view mathematics as an interesting concentration in their liberal arts education, even though they plan to enter professional fields such as medicine or law.

The Mathematics program gives majors and minors a broad introduction to the subject. To fit students' varying academic interests and professional goals, the department offers majors in Mathematics, Applied Mathematics, and Mathematical Sciences, along with two joint majors in Mathematics and Computer Science and in Mathematics and Economics. Majors are encouraged to complete additional work (perhaps even a minor or a second major) in other related areas.

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## Faculty

### Chair

#### John Shareshian

Professor  
PhD, Rutgers University  
Algebraic and topological combinatorics

### Director of Graduate Studies

#### Gregory Knese

Professor  
PhD, Washington University  
Complex function theory; operators; harmonic analysis

### Director of Undergraduate Studies

#### Ari Stern

Professor  
PhD, California Institute of Technology  
Geometric numerical analysis; computational mathematics

### Associate Director of Undergraduate Studies

#### Blake Thornton

Teaching Professor  
PhD, University of Utah  
Geometric topology

### Department Faculty

#### Roya Beheshti Zavareh

Professor  
PhD, Massachusetts Institute of Technology  
Algebraic geometry

#### Alan Chang

Assistant Professor  
PhD, University of Chicago  
Geometric measure theory; harmonic analysis

#### Quo-Shin Chi

Professor  
PhD, Stanford University  
Differential geometry

#### Lawrence Conlon

Emeriti Professor  
PhD, Harvard University  
Differential topology

#### Aliakbar Daemi

Assistant Professor  
PhD, Harvard University  
Gauge theory; low-dimensional topology; symplectic geometry

#### Laura Escobar Vega

Associate Professor  
PhD, Cornell University  
Combinatorics; algebraic geometry

#### Renato Feres

Professor  
PhD, California Institute of Technology  
Differential geometry; dynamical systems

#### Steven Frankel

Associate Professor  
PhD, University of Cambridge  
Geometric topology; dynamics

#### Ron Freiwald

Emeriti Professor  
PhD, University of Rochester  
General topology

#### Andrew Walton Green

William Chauvenet Postdoctoral Lecturer  
PhD, Clemson University  
Harmonic analysis; partial differential equations

**Gary R. Jensen**

Emeriti Professor  
PhD, University of California, Berkeley  
Differential geometry

**Silas Johnson**

Senior Lecturer  
PhD, University of Wisconsin–Madison  
Algebraic number theory; arithmetic statistics

**Matt Kerr**

Professor  
PhD, Princeton University  
Algebraic geometry; Hodge theory

**Steven G. Krantz**

Professor  
PhD, Princeton University  
Several complex variables; geometric analysis

**N. Mohan Kumar**

Emeriti Professor  
PhD, Bombay University  
Algebraic geometry; commutative algebra

**Wanlin Li**

Assistant Professor  
PhD, University of Wisconsin–Madison  
Number theory; arithmetic geometry

**Henri Martikainen**

Associate Professor  
PhD, University of Helsinki, Finland  
Harmonic analysis; geometric measure theory

**John E. McCarthy**

Spencer T. Olin Professor of Mathematics  
PhD, University of California, Berkeley  
Analysis; operator theory; one and several complex variables

**Minh Nguyen**

Postdoctoral Lecturer  
PhD, University of Arkansas  
Gauge theory; low dimensional topology

**Charles Ouyang**

Assistant Professor  
PhD, Rice University  
(Higher) Teichmüller theory; Riemann surfaces; harmonic maps and minimal surfaces

**Martha Precup**

Associate Professor  
PhD, University of Notre Dame  
Applications of Lie theory to algebraic geometry and the related combinatorics

**Donsub Rim**

Assistant Professor  
PhD, University of Washington  
Applied mathematics

**Rachel Roberts**

Elinor Anheuser Professor of Mathematics  
PhD, Cornell University  
Low-dimensional topology

**Richard Rochberg**

Emeriti Professor  
PhD, Harvard University  
Complex analysis; interpolation theory

**Angel Roman**

Postdoctoral Lecturer  
PhD, Pennsylvania State University  
Representation theory; operator algebras

**Jesus Sanchez**

Postdoctoral Lecturer  
PhD, Pennsylvania State University  
Noncommutative index theory; cyclic cohomology; spin Riemannian geometry; high-dimensional gauge theory

**Karl Schaefer**

Lecturer  
PhD, University of Chicago  
Algebraic number theory

**Jack Shapiro**

Emeriti Professor  
PhD, City University of New York  
Algebraic K-theory

**Edward Spitznagel**

Emeriti Professor  
PhD, University of Chicago  
Statistics; statistical computation; application of statistics to medicine

**Yanli Song**

Associate Professor  
PhD, Pennsylvania State University  
Noncommutative geometry; symplectic geometry; representation theory

**Xiang Tang**

Professor  
PhD, University of California, Berkeley  
Symplectic geometry; noncommutative geometry; mathematical physics

**Joel Villatoro**

Postdoctoral Lecturer  
PhD, University of Illinois at Urbana-Champaign  
Differential geometry; Poisson geometry; singular spaces

**Brett Wick**

Professor  
PhD, Brown University  
Complex analysis; harmonic analysis; operator theory; several complex variables

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**Mladen Victor Wickerhauser**

Professor  
PhD, Yale University  
Harmonic analysis; wavelets; numerical algorithms for data compression

**Edward N. Wilson**

Emeriti Professor  
PhD, Washington University  
Harmonic analysis; differential geometry

**David Wright**

Emeriti Professor  
PhD, Columbia University  
Affine algebraic geometry; polynomial automorphisms

**Jay Yang**

Postdoctoral Lecturer  
PhD, University of Wisconsin–Madison  
Commutative algebra; algebraic geometry

## Majors

- Applied Mathematics Major
- Mathematical Sciences Major
- Mathematics and Computer Science Major
- Mathematics and Economics Major
- Mathematics Major

## Minors

- Mathematics Minor

## Courses

Visit online course listings to view semester offerings for L24 Math.

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**L24 Math 100 Foundations for Calculus**

A limited enrollment class for students planning to take calculus but who need additional precalculus preparation. The course aims to build both the technical skills and the conceptual understanding needed to succeed in calculus. Course emphasizes links between the graphical, numeric, and algebraic viewpoints. A variety of approaches are used to present the material. Prerequisites: 2 years of high school algebra and a course in geometry (or the equivalent).

Credit 3 units. A&S IQ: NSM

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**L24 Math 109 Mathematics and Music**

An elementary introduction to the connections between mathematics and musical sound. Review of integers, ratios, prime numbers, functions, rationality, exponents, logarithms, trigonometry. Review of scales, clefs, key signatures, intervals, time signatures. Frequency and pitch. The connection between intervals and logarithms. Tuning and temperament, just intonation. Scales and modular arithmetic. The mathematics of harmony; the sound of the low prime numbers and their roles in harmony. Harmonics, partials and overtones. Numerical integration and basic Fourier analysis. The nature of complex tones. Analysis of instrument sounds. Human vowels and formants. Prerequisites: 2 years of high school algebra, and trigonometry.

Credit 3 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 131 Calculus I**

Derivatives of algebraic, trigonometric, and transcendental functions, techniques of differentiation, Mean Value Theorem, applications of the derivative. The definite integral and Fundamental Theorem of Calculus. Areas. Simpler integration techniques. Prerequisites: high school algebra and precalculus, including trigonometry.

Credit 3 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 131E Calculus I Extended**

Math 131E covers the same content as Math 131 but includes the additional review of precalculus concepts integrated throughout the semester. It is aimed at students whose precalculus skills are not yet fully developed. By the end of this course, students should be ready to enroll in Math 132.

Credit 4 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 132 Calculus II**

Continuation of Math 131. A brief review of the definite integral and Fundamental Theorem of Calculus. Techniques of integration, applications of the integral, sequences and series, Taylor polynomials and series, and some material on differential equations. Prerequisite: Math 131 or a B or better in a one-year high school calculus course, or permission of the department.

Credit 3 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 203 Honors Mathematics I**

This is the first half of a one-year calculus sequence for first year student with a strong interest in mathematics with an emphasis on rigor and proofs. The course begins at the beginning but assumes the students have already studied the material from a more "mechanical" view. Students who complete both semesters will have complete the material Calc III and other topics that may let them move through the upper level math curriculum more quickly. Sets, functions, real numbers, and methods of proof. The Riemann-Darboux integral, limits and continuity, differentiation, and the fundamental theorems of calculus. Sequences and series of real numbers and of functions. Vector spaces and linear maps. Prerequisite: Score of 5 on the A.P Calculus Exam, BC version, or the equivalent.

Credit 4 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 204 Honors Mathematics II**

Matrices, linear systems, and determinants. Eigenvalues and eigenvectors, diagonalization, and the spectral theorem. Scalar and vector fields, differential and integral calculus of several variables, and the fundamental theorems of Green, Gauss, and Stokes. Restricted to first year students who have completed Math 203 in the fall semester. Math 204 can replace Math 233 in major/minor requirements.

Credit 4 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 217 Differential Equations**

Introduction to ordinary differential equations: first-order equations, linear equations, systems of equations, series solutions, Laplace transform methods, numerical solutions. Prerequisite: Math 233 (or Math 233 concurrently).

Credit 3 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 220 Finite Mathematics**

Topics from discrete mathematics will be explored with an emphasis on problem-solving and methods of proofs. Modules on counting; combinatorial tools; binomial coefficients and Pascal's triangle; Fibonacci numbers; combinatorial probability; integers, divisors and primes; and graphs will be covered as well as additional topics as

time permits. Addressed mainly to college freshmen and sophomores; it would also be suitable to advanced high school students with an interest in mathematics. Prerequisites: A good understanding of high school mathematics.

Credit 3 units. A&S IQ: NSM, AN Art: NSM

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**L24 Math 233 Calculus III**

Multivariable calculus. Topics include differential and integral calculus of functions of two or three variables: vectors and curves in space, partial derivatives, multiple integrals, line integrals, vector calculus at least through Green's Theorem. Prereq: Math 132, or a score of 4-5 on the Advanced Placement Calculus Exam (BC version).

Credit 3 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 308 Mathematics for the Physical Sciences**

Continuation of Math 233 emphasizing topics of interest in the physical sciences. Topics in multivariable and vector calculus (div, grad, curl); line, surface integrals and connections to electromagnetism; Fourier series and integrals; boundary value problems (diffusion and wave equations); additional topics if time permits. Students may not receive credit toward a math major or minor for both Math 308 and Math 318. Prerequisite: Math 233 and 217, or permission of instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM BU: SCI

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**L24 Math 309 Matrix Algebra**

An introductory course in linear algebra that focuses on Euclidean  $n$ -space, matrices and related computations. Topics include: systems of linear equations, row reduction, matrix operations, determinants, linear independence, dimension, rank, change of basis, diagonalization, eigenvalues, eigenvectors, orthogonality, symmetric matrices, least square approximation, quadratic forms. Introduction to abstract vector spaces. Prerequisite: Math 132.

Credit 3 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 310 Foundations for Higher Mathematics**

Introduction to the rigorous techniques used in more advanced mathematics. Topics include propositional logic, use of quantifiers, set theory, methods of proof and disproof (counterexamples), foundations of mathematics. Use of these tools in the construction of number systems, and in other areas such as elementary number theory, combinatorial arguments, and elementary proofs in analysis. Prerequisite: Math 233.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 310W Foundations For Higher Mathematics With Writing**

Introduction to the rigorous techniques used in more advanced mathematics. Topics include basic logic, set theory, methods of proof and counterexamples, foundations of mathematics, construction of number systems, counting methods, combinatorial arguments and elementary analysis. At least 3 papers will be required, which with at least one revision. Prerequisite: Math 233.

Credit 3 units. A&S IQ: NSM, WI Art: NSM

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**L24 Math 312 Differential Equations and Dynamical Systems**

Qualitative theory of ordinary differential equations. Picard's existence and uniqueness theorem, the phase plane, Poincare-Bendixon theory, stationary points, attractors and repellers, graphical methods. Physical applications, including chaos, are indicated. Prerequisite: Math 217.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 318 Introduction to Calculus of Several Variables**

Selected topics for functions of several variables involving some matrix algebra and presented at a level of rigor intermediate between that of Calculus III and higher level analysis courses. Students may not receive credit toward a mathematics major or minor for both Math 308 and 318. Prerequisites: Math 233 and Math 309. Math 310 is recommended but not required.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 331 Algebraic Systems**

Polynomials, binomial expansions, factoring, rings (integers and polynomials), unique factorization, and other topics relevant to the high school curriculum. Designed for future secondary school teachers and other students looking for a course in algebra at a less abstract level than Math 430. Prerequisite: Math 310 or permission of instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 350 Topics in Applied Mathematics**

Topics change with each offering of the course. Past topics have included "Mathematics and Multimedia," "The Mathematics and Chemistry of Reaction-Diffusion Systems", "Mathematical Biology," and "Simulation Analysis of Random Processes" and "Introduction to Monte Carlo Methods." Prerequisites will vary, but always include at least Math 233, Math 309 and basic programming skills in some language.

Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 370 Introduction to Combinatorics**

Basics of enumeration (combinations, permutations and enumeration of functions between finite sets), generating functions; the inclusion-exclusion principle, partition theory and introductory graph theory. As time permits, additional topics may include Ramsey's Theorem, probabilistic methods in combinatorics and algebraic methods in combinatorics. Prerequisites: Math 132, 309 and 310, or permission of the instructor.

Credit 3 units. A&S IQ: NSM, AN Arch: NSM Art: NSM

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**L24 Math 371 Graph Theory**

Introduction to graph theory including the basic definitions and theorems and some more advanced topics which drive much current research in graph theory: Ramsey's Theorem, random graph theory and, if time permits, Szemerédi's regularity lemma. Graphs will be studied as abstract objects; however graph theory is also of interest to applied mathematicians because graphs are natural models for networks (social, electric,...). Prerequisite: Math 310 or a roughly equivalent course, or permission of instructor. Students should know what a proof is and how to produce one. Some informal understanding of probability will be helpful, but students need not have taken a probability course.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 401 Honors Seminar in Math**

Credit 3 units.

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**L24 Math 407 An Introduction to Differential Geometry**

A study of properties of curves and surfaces in 3-dimensional Euclidean space. The course is essentially a modern recounting of a seminal paper of Gauss. Prerequisites: Math 233, Math 309, Math 310.

Credit 3 units. A&S IQ: NSM

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**L24 Math 410 Introduction to Fourier Series and Integrals**

The basic theory of Fourier series and Fourier integrals including different types of convergence. Applications to certain differential equations. Prerequisites: Math 4111 or permission of instructor.

Credit 3 units. A&S IQ: NSM

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**L24 Math 4111 Introduction to Analysis**

The real number system and the least upper bound property; metric spaces (completeness, compactness, and connectedness); continuous functions (in  $\mathbb{R}^n$ ; on compact spaces; on connected spaces);  $C(X)$  (pointwise and uniform convergence; Weierstrass approximation theorem); differentiation (mean value theorem; Taylor's theorem); the contraction mapping theorem; the inverse and implicit function theorems. Prerequisite: Math 310 or permission of instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 4121 Introduction to Lebesgue Integration**

Riemann integration; measurable functions; measures; Lebesgue measure; the Lebesgue integral; integrable functions;  $L^p$  spaces; modes of convergence; decomposition of measures; product measures. Prerequisite: Math 4111 or permission of the instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 415 Partial Differential Equations**

Introduction to the theory of PDE's with applications to selected classical problems in physics and engineering. Linear and quasilinear first order equations, derivation of some of the classical PDE's of physics, and standard solution techniques for boundary and initial value problems. Preliminary topics such as orthogonal functions, Fourier series, and variational methods introduced as needed. Prerequisites: Math 217 or 312, Math 309, and Math 310, or permission of instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 416 Complex Variables**

Analytic functions, elementary functions and their properties, line integrals, the Cauchy integral formula, power series, residues, poles, conformal mapping and applications. Prereq: Math 310 and (Math 318 or Math 4111), or permission of instructor.

Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 4171 Topology I**

An introduction to the most important ideas of topology. Course includes necessary ideas from set theory, topological spaces, subspaces, products and quotients, compactness and connectedness. Some time is also devoted to the particular case of metric spaces (including topics such as separability, completeness, completions, the Baire Category Theorem, and equivalents of compactness in metric spaces). Prerequisite: Math 4111 or permission of instructor.

Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 4181 Topology II**

A continuation of Math 4171 featuring more advanced topics in topology. The content may vary with each offering. Prerequisite: Math 4171, or permission of instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 429 Linear Algebra**

This course is an introduction to the linear algebra of finite-dimensional vector spaces. It includes systems of equations, matrices, determinants, inner product spaces, and spectral theory. Prerequisite: Math 310 or permission of instructor. Math 309 is not an explicit prerequisite, but students should already be familiar with such basic topics from matrix theory as matrix operations, linear systems, row reduction, and Gaussian elimination. (Material on these topics in early chapters of the text will be covered very quickly.)

Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 430 Modern Algebra**

Introduction to groups, rings, and fields. Includes permutation groups, group and ring homomorphisms, field extensions, connections with linear algebra. Prerequisite: Math 310, Math 429 or permission of the instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 4351 Number Theory and Cryptography**

The course will cover many of the basics of elementary number theory, providing a base from which to approach modern algebra, algebraic number theory and analytic number theory. It will also introduce one of the most important real-world applications of mathematics, namely the use of number theory and algebraic geometry in public key cryptography. Topics from number theory involve divisibility (Euclidean algorithm, primes, Fundamental Theorem of Arithmetic), congruences (modular arithmetic, Chinese Remainder Theorem, primality testing and factorization). Topics from cryptography will include RSA encryption, Diffie-Hellman key exchange and elliptic curve cryptography. Topics about algebraic numbers may be included if time permits. Prerequisites: Math 233, 309 and 310 (or permission of instructor)

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 436 Algebraic Geometry**

Introduction to affine and projective algebraic varieties, the Zariski topology, regular and rational mappings, simple and singular points, divisors and differential forms, genus, the Riemann-Roch theorem. Prerequisites: Math 310, 429, and 430, or permission of the instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 444 The Mathematics of Quantum Theory**

An introduction to the mathematical foundations of quantum theory aimed at advanced undergraduate/beginning graduate students in Mathematics and Engineering, although students from other disciplines are equally welcome to attend. Topics include: the mathematical postulates of quantum theory and simple physical systems, spectral theory of self-adjoint operators, rudiments of Lie groups, Lie algebras and unitary group representations, elements of quantum probability and quantum information theory. Prerequisites: Linear algebra at the level of Math 429 or equivalent, multivariate calculus at the level of Math 318, and basic probability theory at the undergraduate level such as Math 493 or instructor's permission.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 449 Numerical Applied Mathematics**

Computer arithmetic, error propagation, condition number and stability; mathematical modeling, approximation and convergence; roots of functions; calculus of finite differences; implicit and explicit methods for initial value and boundary value problems; numerical integration; numerical solution of linear systems, matrix equations, and eigensystems; Fourier transforms; optimization. Various software packages may be introduced and used. Prerequisites: Math 217 or 312, Math 309, Math 310 and CSE 131 (or other computer background with permission of the instructor).

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 450 Topics in Applied Mathematics**

Topic may vary with each offering of the course. Prerequisite: CSE 131 and, Math 449, or permission of the instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

**L24 Math 456 Topics in Financial Mathematics**

An introduction to the principles and methods of financial mathematics, with a focus on discrete-time stochastic models. Topics include no-arbitrage pricing of financial derivatives, risk-neutral probability measures, the Cox-Ross-Rubenstein and Black-Scholes-Merton options pricing models, and implied volatility. Prerequisites: Math 233, Math 3200, Math 310 or permission of instructor.

Credit 3 units. A&S IQ: NSM Arch: NSM Art: NSM

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**L24 Math 470 Topics in Graph Theory**

Credit 3 units. A&S IQ: NSM

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**L24 Math 493C Probability**

Mathematical theory and application of probability at the advanced undergraduate level; a calculus based introduction to probability theory. Topics include the computational basics of probability theory, combinatorial methods, conditional probability including Bayes' theorem, random variables and distributions, expectations and moments, the classical distributions, and the central limit theorem. permission of the instructor. Prerequisites: Math/SDS 3200 and Math 233.

Same as L87 SDS 493

Credit 3 units. A&S IQ: NSM Art: NSM

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**L24 Math 495C Stochastic Processes**

Content varies with each offering of the course. Past offerings have included such topics as random walks, Markov chains, Gaussian processes, empirical processes, Markov jump processes, and a short introduction to martingales, Brownian motion and stochastic integrals. Prerequisites: Math 309; Math/SDS 493 or Math/SDS 3211.

Same as L87 SDS 495

Credit 3 units. A&S IQ: NSM Art: NSM

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